The Impact of Cognitive Activation Practices on Enhancing Student Mathematical Reasoning in Mathematical Literacy: A Causal Comparative Study Based on PISA 2022

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Abstract: This study aimed to describe and compare cognitive activation practices designed to enhance students' mathematical reasoning in Saudi Arabia (KSA), the United States (USA), the United Arab Emirates (UAE), and Singapore. It also explored the impact of these practices on students' mathematical literacy development in each country. Using a causal-comparative methodology, the research examined student scores from the PISA 2022 mathematical literacy assessments, alongside responses to specific questionnaire items on teachers' cognitive activation practices aimed in fostering mathematical reasoning.

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The findings revealed statistically significant differences in the implementation of cognitive activation practices across the countries studied. Saudi Arabia demonstrated significantly lower levels of these practices compared to the other nations, while the UAE exhibited significantly higher levels. No statistically significant differences were observed between the USA and Singapore in this regard. Moreover, the results indicated that all cognitive activation practices had a predictive effect on the variance in mathematical literacy among students in each country\. Notably, certain practices exhibited a negative impact on mathematical literacy in some countries while contributing positively in others.

Keywords: teaching quality, effective teaching, cognitive activation, mathematical literacy, international assessments.

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Introduction

The Kingdom of Saudi Arabia (KSA) places significant emphasis on developing human capabilities and preparing globally competitive students. Therefore, Saudi curricula reflect the latest educational trends according to the best international practices and recent literature in mathematics education. To meet its aspirations, KSA has participated in relevant international assessments, including the Program for International Student Assessment (PISA), which aims to measure students' ability to apply their knowledge and skills in reading, mathematics, and science to address real-life challenges.

In this context, PISA offers a practical framework for the necessary mathematical literacy in all its assessments, which are conducted every three years, considering the development of knowledge and the requirements of the era. In its latest framework for the 2022 PISA assessments, a special importance was placed on reasoning and providing students with opportunities for mathematical thinking and deductive reasoning. Additionally, it is crucial to equip students with fundamental mathematical tools, concepts, and a conceptual framework to navigate the quantitative dimensions of life in the 21st century. Effective teaching prepares students to think efficiently in unfamiliar situations (OECD, 2023; Tourón et al., 2019).

Several studies have highlighted the importance of teachers and their practices in influencing learning outcomes (Tourón et al., 2019; Demir, 2018; Hwang, Choi et al., 2018; Elliott et al., 2018; Le Donné et

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al., 2016; Yi & Lee, 2017). It should be noted that mathematical illiteracy may not solely stem from the content itself, but rather from the pedagogical approach utilized in teaching it. Moreover, traditional mathematics education tends to be overly formal, less intuitive, more abstract, less contextual, more symbolic, and less realistic compared to an education that expands students' thinking and develops their mathematical literacy (Martain, 2007).

From this perspective, the National Council of Teachers of Mathematics (NCTM) has presented a set of practices for mathematics education that aims to deepen mathematical knowledge (2014/2019). These practices emphasize the importance of establishing clear educational objectives to facilitate meaningful learning. Additionally, it is crucial to engage students in tasks that enhance their reasoning abilities, encourage the use and connection of mathematical representations, foster effective dialogue to deepen students' understanding and construction of ideas, pose purposeful questions to assess and develop reasoning skills, establish procedural fluency and conceptual understanding, and support productive struggle in mathematics education.

Theoretical Background

One key element in advanced practices for mathematical literacy is the principles of Realistic Mathematics Education theory (RME). This theory emphasizes the necessity of learning mathematics as a human activity, rather than a ready-made product. It begins by using real-world

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contexts as a foundation for learning and incorporates visual models that bridge the gap between abstract concepts and real-world applications This approach encourages students to develop their problem-solving strategies instead of memorizing rules and procedures, integrates mathematical communication as an essential part of lessons, and ensures connections with other disciplines through meaningful contextual problems (Martain, 2007).

Hwang et al. (2018) argue that educational practices play a crucial role in reducing the knowledge gap resulting from disparate socioeconomic conditions across various countries. By analyzing data from PISA 2012-2015 assessments, they concluded that student-centered education could potentially narrow the mathematics achievement gap.

Additionally, teaching quality is a crucial factor linked to learning experiences. Consequently, numerous researchers have investigated mathematics teachers' behaviors to identify dimensions that characterize effective instruction (Bolhuis, 2003; Klieme et al., 2009; and Baumert et al.,2010). However, defining quality teaching is complex, as the concept reflected in teachers' behaviors is multifaceted and diverse, with students potentially benefiting from different instructional practices (Yi & Lee, 2017).

Nevertheless, certain characteristics and behaviors can be identified as critical factors for high-quality mathematics education. According to a theoretical model developed by Klieme et al. (2009),

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three fundamental dimensions of teaching quality were identified: cognitive activation, classroom management, and supportive climate.

Cognitive activation refers to any pedagogical practice that encourages students to deepen their knowledge and engage in constructing and reflecting on higher-order thinking (Klieme et al., 2009: p.140-141). Figure 1 illustrates the theoretical model of teaching quality dimensions as per Klieme et al. (2009, p. 140) which has been adopted as a behavioral framework for measuring mathematics teachers' instructional quality in PISA and TALIS studies (Yi & Lee, 2017).

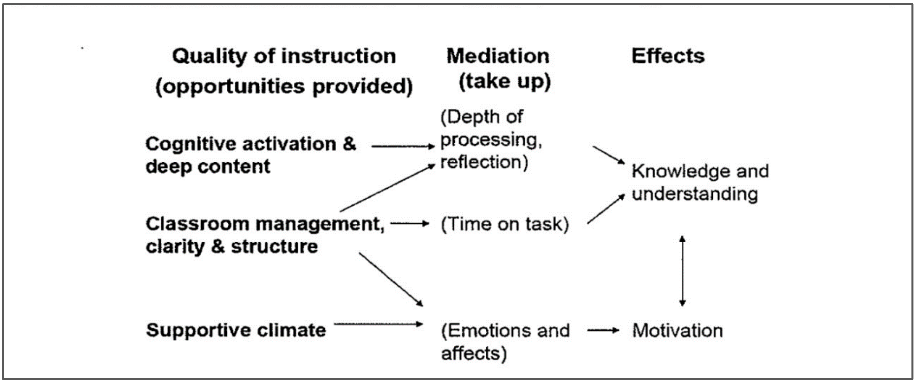

Figure 1: Theoretical model of teaching quality dimensions according to Klieme et al.

In PISA 2022 assessments, elements reflecting teacher behavior in instructional practices were integrated into student questionnaires under the three dimensions of Klieme et al.'s (2009) theoretical model. These dimensions are cognitive activation in mathematics (enhancing logic), cognitive activation in mathematics (encouraging mathematical thinking), and disciplinary climate in mathematics (classroom management) (OECD, n.d., p. 21).

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The elements of teacher behavior in the cognitive activation dimension for enhancing logic align to some extent with the visions and ideas presented by NCTM and with components of RME.

Research Objectives

This study emphasizes that the data collected in such investigations are primarily descriptive, which, alone, does not provide an understanding of what might be successful in other national contexts. However, it enriches contextually embedded environmental analyses by establishing descriptive foundations for future research studies. According to Elliott et al. (2018), research can be conducted using three different approaches: descriptive, interpretive, and theoretical.

Consequently, the main objective of this research is to describe and compare cognitive activation practices for enhancing mathematical reasoning in KSA, the USA, UAE, and Singapore. Additionally, it aims to explore the contribution of these practices to the development of mathematical literacy in each country.

Research Significance

The significance of this research is highlighted in two aspects:

1. The importance of international assessments, as they provide rich data for diagnosing educational systems and comparing them with other systems.
2. The current research focuses on an aspect of effective teaching practices related to cognitive activation for enhancing mathematical

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reasoning according to the student questionnaires in PISA 2022 assessments. The findings of this research will guide the development of effective teaching practices.

Research Questions

1. Are there statistically significant differences in cognitive activation practices to enhance mathematical reasoning among students in KSA, the USA, UAE, and Singapore?

2. To what extent do cognitive activation practices contribute to the development of mathematical literacy among students in KSA, the USA, UAE, and Singapore?

Research Limitations

This research is limited to studying cognitive activation practices using student questionnaires in PISA 2022 assessments. Furthermore, it is limited to analyzing the responses of Saudi, American, Emirati, and Singaporean samples in the PISA 2022 assessments.

Definition of Terms

Cognitive Activation Practices for Enhancing Mathematical Reasoning:

Conceptually, this term refers to pedagogical practices that encourage students to deepen their knowledge and engage in higher-order thinking, such as constructing and reflecting (Klieme et al., 2009: 140-141).

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Procedurally, it refers to the practices listed in the student questionnaire that describe teacher behavior in nine items under the identifier (COGACRCO) (OECD, Report 19 Scaling, n.d.), as expressed through the research sample's responses to these items.

Literature Review

The impact of teaching practices on student performance in mathematics has been extensively researched and debated in recent years. A comprehensive review of the literature revealed various approaches to understanding and measuring the effectiveness of different instructional strategies, particularly in the context of international assessments. For example, a report by the OECD's Directorate for Education explored the relationship between teaching strategies and student learning outcomes in eight countries participating in PISA assessments. The study examined three teaching strategies: active learning, cognitive activation, and teacher-directed instruction. The findings indicated a strong correlation between the use of cognitive activation strategies and student achievement (Le Donné et al., 2016).

Additionally, educational literature has addressed effective teaching practices according to these aspects. Most studies have, in one way or another, concluded that it is necessary to distinguish between cognitive activation and student-oriented teacher behavior. This is because the latter alone may not contribute to effective learning unless deep learning occurs as a form of knowledge activation strategy. Furthermore, some studies revealed a positive effect of cognitive

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activation on learner performance, while reporting a negative effect of student-oriented teacher behavior (Demir, 2018; Le Donné et al., 2016; Yi & Lee, 2017).

On the contrary, a study by Tourón et al. (2019) on a Spanish sample in PISA 2012 assessments showed ambiguous results regarding the impact of cognitive activation on performance. When examining items in detail, results showed negative correlations for some items and positive ones for others. Some item results indicated that moderate use resulted in the maximum value, not extreme values. Consequently, the results raise questions and ambiguous contradictions: can the commonly used elements of cognitive activation have a detrimental effect? How can "encouraging thinking about problems" or "having students apply what they've learned," for instance, negatively impact performance?

For many countries, student performance serves as an important indicator of education system quality. Therefore, participating in international assessments such as PISA is crucial. Locally, KSA participated in these assessments: PISA 2018 and 2022 assessments. The average results in 2022 increased by 16 points in mathematics compared to 2018. However, only 30% of students were at Level 2, which refers to students' ability to interpret and recognize how to represent a relatively simple situation. No student reached the advanced level (Level 5 or 6), despite students reporting high support from their teachers in the questionnaire. 76% reported that in most lessons, teachers show interest

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in every student's learning, and 77% reported that teachers provide extra help when students need it (OECD, 2023).

Therefore, there is a need to examine teachers' practices related to cognitive activation to enhance students' mathematical reasoning in KSA and benchmark the findings with practices in the USA (considering both apply the same McGraw-Hill mathematics curriculum series), UAE (due to similar cultural and economic conditions), and Singapore (as one of the highest-performing countries). Additionally, there is a need to explore how these practices contribute to developing mathematical literacy in each country.

Methodology and Sampling

This study employed a causal-comparative methodology to investigate potential factors (cognitive activation practices for enhancing mathematical reasoning) that may contribute to variations in mathematical literacy.

The target population for PISA 2022 assessments consisted of 15-year-old students. The sampling design followed a two-stage stratified sample, where the first-stage sampling unit comprised schools with 15-year-old students, and the second-stage sampling unit consisted of students within those schools (OECD, 6 Sample Design, n.d.).

The sample sizes for each country were as follows:

Saudi Arabia: 6,928 students from 193 schools.

Singapore: 6,606 students from 164 schools.

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United Arab Emirates: 24,600 students from 840 schools.

United States: 4,552 students from 154 schools.

Data Sources

The research utilized two primary data sources from the PISA 2022 assessments, published on the Organization for Economic Co-operation and Development (OECD) website:

1. Students' mathematical literacy test scores from the four target countries.
2. The responses of the sample students to items in the student questionnaire under the identifier (COGACRCO), which relate to cognitive activation practices.

Analysis and Findings

Research Question 1: Are there statistically significant differences in cognitive activation practices for enhancing mathematical reasoning among students in KSA, the USA, UAE, and Singapore?

To address this question, a one-way analysis of variance (ANOVA) was employed to compare the countries based on their level of cognitive activation practices for enhancing mathematical reasoning. The results are presented as follows:

I: Descriptive Statistics: Table 1 illustrates the level of cognitive activation practices for enhancing mathematical reasoning:

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Table 1: Means and Standard Deviations of Countries in Cognitive Activation Practices for Enhancing Mathematical Reasoning

Cognitive Activation Practices	KSA		Singapore		UAE		USA	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
The teacher asked us to solve mathematical problems without performing calculations.	٢,١٣	١,٢٦٣	٢,١٧	١,٢٣٠	٢,٥١	١,٣٩٧	٢,٣٩	١,٣٣٩
The teacher requested that we explain our problem-solving process for a mathematical question.	٣,١٨	١,٣٨٢	٣,٤٠	١,٢١٨	٣,٤٩	١,٢٨٣	٣,٥٣	١,٢٩٢
The teacher asked us to explain the assumptions we made when approaching a mathematical problem.	٢,٨٩	١,٣٤٢	٣,٠١	١,٢٧٢	٣,٢١	١,٢٩٢	٣,٠٦	١,٣٣٥
The teacher required us to articulate our reasoning when solving a mathematical problem.	٣,٠٢	١,٣٥٩	٣,٤٤	١,٢١٥	٣,٤٧	١,٢٦٤	٣,٥٥	١,٢٧٨
The teacher asked us to defend our answers to mathematical questions.	٢,٨٦	١,٣٦٧	٢,٩١	١,٣١٣	٣,٣٢	١,٢٩٠	٣,٢١	١,٣٥٧
The teacher encouraged us to consider the connections between new and previously learned mathematical concepts.	٢,٨٥	١,٣٨٢	٣,١٦	١,٣٠١	٣,١١	١,٣٤٥	٢,٨٠	١,٤٠٨
The teacher encouraged us to explore alternative problem-solving methods beyond those demonstrated in class.	٣,٣٢	١,٤٢٥	٣,٣٨	١,٢٤٩	٣,٤٠	١,٣١٨	٣,١١	١,٤٠٨
The teacher instructed us to persevere when encountering difficulties in mathematical tasks.	٣,٤٤	١,٤٣٤	٣,٩٠	١,١١٨	٣,٦٦	١,٢٨٤	٣,٦٩	١,٢٧٧
The teacher taught us to memorize rules and apply	٣,٦٠	١,٤٤١	٣,٩٠	١,٠٧٢	٣,٧٢	١,٢٦٨	٣,٧٤	١,٢١٩

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Cognitive Activation Practices	KSA		Singapore		UAE		USA	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
them in solving mathematical problems.								
Overall cognitive activation practices	٣,٠٣	١,١٢١	٣,٢٥	٠,٩٣٠	٣,٣٢	١,٠٤٣	٣,٢٣	١,٠٣٠

The descriptive analysis revealed notable variations in cognitive activation practices across the four countries studied. UAE demonstrated the highest mean scores in four practices: Solving mathematics problems without calculation (M = 2.51), explaining assumptions in problem-solving (M = 3.21), defending answers to mathematical questions (M = 3.32), and encouraging alternative problem-solving methods (M = 3.40). On the other hand, the USA demonstrated the highest mean in two practices: explaining the problem-solving process (M = 3.53), and articulating reasoning in problem-solving (M = 3.55).

Singapore showed the highest means in three practices: connecting new and previously learned mathematical concepts (M = 3.16), persevering when facing difficulties in mathematical tasks (M = 3.90) and memorizing and applying rules in problem-solving (M = 3.90). Nevertheless, KSA consistently showed the lowest mean scores across most practices. Overall, the UAE demonstrated the highest level of cognitive activation practices for enhancing mathematical reasoning, with a mean score of 3.32, while Saudi Arabia showed the lowest overall level, with a mean score of 3.03.

II: Differences and Statistical Significance

A one-way analysis of variance (ANOVA) was conducted to examine the differences in cognitive activation practices for enhancing mathematical reasoning in KAS, the USA, the UAE, and Singapore,

Table 2: Results of One-Way ANOVA for Differences in Cognitive Activation Practices for Enhancing Mathematical Reasoning Across Countries.

Cognitive Activation Practices	Source of Variation	Su m of Squared Deviations	Degree of Freedom (df)	Mean of Squared Deviation	F-Statistics	The P=Value of significance Level (Sig.)
The teacher asked us to solve mathematical problems without performing calculations.	Among groups	٥٨١,٠٨٠	٣	١٩٣,٦٩٣	١٠٧,٦١٥	٠,٠٠
	Within groups	٣٧٣٧٤,٢٣٩	٢٠٧٦٥	١,٨٠٠		
	Total	٣٧٩٥٥,٣١٩	٢٠٧٦٨			
The teacher requested that we explain our problem-solving process for a mathematical question.	Among groups	٢٦٩,٥٧٥	٣	٨٩,٨٥٨	٥٤,٠٤٧	٠,٠٠
	Within groups	٣٤٠٤٤,٦٦٩	٢٠٤٧٧	١,٦٦٣		
	Total	٣٤٣١٤,٢٤٤	٢٠٤٨٠			
The teacher asked us to explain the assumptions we made when approaching a mathematical problem.	Among groups	٣١٢,٦٣٠	٣	١٠٤,٢١٠	٦١,٥٤٢	٠,٠٠
	Within groups	٣٤٨٨٢,٤٦٥	٢٠٦٠٠	١,٦٩٣		
	Total	٣٥١٩٥,٠٩٤	٢٠٦٠٣			
The teacher required us to articulate our reasoning when solving a mathematical problem.	Among groups	٥٨٧,٨٤٧	٣	١٩٥,٩٤٩	١٢٠,٩١٢	٠,٠٠
	Within groups	٣٣٧٠٠,١٧١	٢٠٧٩٥	١,٦٢١		
	Total	٣٤٢٨٨,٠١٨	٢٠٧٩٨			
The teacher asked us to defend our answers to mathematical questions.	Among groups	٨٤٨,٤٧٠	٣	٢٨٢,٨٢٣	١٦٣,٩٣١	٠,٠٠
	Within groups	٣٥٦٩٢,١٥٨	٢٠٦٨٨	١,٧٢٥		
	Total	٣٦٥٤٠,٦٢٨	٢٠٦٩١			
The teacher encouraged us to consider the connections between new and previously learned mathematical concepts.	Among groups	٣٤٤,٩٠٦	٣	١١٤,٩٦٩	٦٣,٠٥٧	٠,٠٠
	Within groups	٣٧٧٩٨,٠٢٩	٢٠٧٣١	١,٨٢٣		
	Total	٣٨١٤٢,٩٣٦	٢٠٧٣٤			
The teacher encouraged us to explore alternative problem-solving methods beyond those demonstrated in class.	Among groups	١٦٠,١٥١	٣	٥٣,٣٨٤	٣٠,٠٠٢	٠,٠٠
	Within groups	٣٦٨٥٣,٧٥٥	٢٠٧١٢	١,٧٧٩		
	Total	٣٧٠١٣,٩٠٧	٢٠٧١٥			
The teacher instructed us to persevere when encountering difficulties in mathematical tasks.	Among groups	٣٧٣,٤٢٢	٣	١٢٤,٤٧٤	٧٥,٧٨٨	٠,٠٠
	Within groups	٣٤٠٩٧,٦٠٥	٢٠٧٦١	١,٦٤٢		
	Total	٣٤٤٧١,٠٢٧	٢٠٧٦٤			
The teacher taught us to memorize rules and apply them in solving mathematical problems.	Among groups	١٥٧,٢٩٩	٣	٥٢,٤٣٣	٣٢,٩٦٦	٠,٠٠
	Within groups	٣٢٨٢٣,٠٩١	٢٠٦٣٧	١,٥٩٠		
	Total	٣٢٩٨٠,٣٩٠	٢٠٦٤٠			

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Cognitive Activation Practices	Source of Variation	Su m of Squared Deviations	Degree of Freedom (df)	Mean of Squared Deviation	F-Statistics	The P=Value of significance Level (Sig.)
Overall cognitive activation practices	Among groups	٣٩٦,٧١٦	٣	١٣٢,٢٣٩	١٢٣,١٦٧	٠,٠٠
	Within groups	٤٠٤٤٨,٧٣٠	٣٧٦٧٤	١,٠٧٤		
	Total	٤٠٨٤٥,٤٤٦	٣٧٦٧٧			

The one-way ANOVA results revealed statistically significant differences among the countries in cognitive activation practices for enhancing mathematical reasoning. These differences were observed across all nine practices and in the overall dimension.

To determine the specific nature of these differences, Scheffe's post-hoc multiple comparison tool was employed, as shown in Table 3.

<u>III: post-hoc multiple comparisons</u>

Table 3: Comparison of Mean Cognitive Activation Practices by Country

Practices	Country	Singapore	UAE	USA
The teacher asked us to solve mathematical problems without performing calculations.	KSA	٠,٠٤٠-	*٠,٣٨٦-	*٠,٢٥٩-
	USA	*٠,٢١٩	*٠,١٢٧-	--
	UAE	*٠,٣٤٦	--	--
The teacher requested that we explain our problem-solving process for a mathematical question.	KSA	*٠,٢٢٤-	*٠,٣٠٩-	*٠,٣٥٣-
	USA	*٠,١٢٩	٠,٠٤٤	--
	UAE	*٠,٨٥	--	--
The teacher asked us to explain the assumptions we made when approaching a mathematical problem.	KSA	*٠,١١٦-	*٠,٣١٧-	*٠,١٦٧-
	USA	٠,٠٥١	*٠,١٥١-	--
	UAE	*٠,٢٠١	--	--
The teacher required us to articulate our reasoning when solving a mathematical problem.	KSA	*٠,٤١٥-	*٠,٤٤٨-	*٠,٥٢٧-
	USA	*٠,١١٢	٠,٠٧٩	--
	UAE	٠,٠٣٣	--	--
The teacher asked us to defend our answers to mathematical questions.	KSA	٠,٠٥٢-	*٠,٤٦٧-	*٠,٣٥٦-
	USA	*٠,٣٠٤	*٠,١١١-	---
	UAE	*٠,٤١٥	--	--

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Practices	Country	Singapore	UAE	USA
The teacher encouraged us to consider the connections between new and previously learned mathematical concepts.	KSA	*٠,٣٠٧-	*٠,٢٥٣-	٠,٠٥٢
	USA	*٠,٣٦٠-	*٠,٣٠٥-	---
	UAE	٠,٥٥-	--	--
The teacher encouraged us to explore alternative problem-solving methods beyond those demonstrated in class.	KSA	٠,٠٥٧-	*٠,٠٨٠-	*٠,٢٠٨
	USA	*٠,٢٦٤-	*٠,٢٨٧-	--
	UAE	٠,٠٢٣	--	--
The teacher instructed us to persevere when encountering difficulties in mathematical tasks.	KSA	*٠,٤٦١-	*٠,٢١٧-	*٠,٢٥١-
	USA	*٠,٢١٠-	٠,٠٣٤	--
	UAE	*٠,٢٤٤-	--	--
The teacher taught us to memorize rules and apply them in solving mathematical problems.	KSA	*٠,٢٩٧-	*٠,١٢٢-	*٠,١٤٢-
	USA	*٠,١٥٦-	٠,٠١٩	--
	UAE	*٠,١٧٥-	--	--
Overall cognitive activation practices	KSA	*٠,٢١٨-	*٠,٢٨٩-	*٠,١٩٩-
	USA	٠,٠٢٠-	*٠,٠٩١-	--
	UAE	*٠,٠٧١	--	--
Note: * indicates statistical significance at p < .05				

The Scheffe post-hoc multiple comparison results revealed significant variations in cognitive activation practices across the countries studied. KSA consistently demonstrated the lowest levels of implementation across most practices, with statistically significant differences compared to other countries. On the other hand, the USA showed significantly higher implementation than Singapore in several practices: solving mathematics problems without calculation, explaining problem-solving processes, articulating reasoning in problem-solving, and defending mathematical answers. However, the USA scored significantly lower than Singapore in the other activation practices namely: connecting new and old mathematical concepts, encouraging

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alternative problem-solving methods, persevering through mathematical difficulties, and memorizing and applying rules in problem-solving.

Additionally, the UAE exhibited significantly higher implementation than Singapore in solving mathematics problems without calculation, explaining problem-solving processes, explicating assumptions in problem-solving, and defending mathematical answers. Nevertheless, the UAE scored significantly lower than Singapore in connecting new and old mathematical concepts, persevering through mathematical difficulties, and memorizing and applying rules in problem-solving. Furthermore, the UAE demonstrated significantly higher implementation than the USA in solving mathematical problems without calculation, explicating assumptions in problem-solving, defending mathematical answers, connecting new and old mathematical concepts, and encouraging alternative problem-solving methods.

Overall, KSA consistently showed the lowest implementation of these practices, with statistically significant differences compared to other countries. On the other hand, the UAE demonstrated the highest implementation, significantly differing from other countries. Additionally, no significant overall differences were observed between the United States and Singapore.

Research Question 2: To what extent do cognitive activation practices contribute to the development of mathematical literacy among students in KSA, the USA, UAE, and Singapore?

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To address this question, multiple regression analysis was conducted using the IDB Analyzer. Table 4 presents the B and Beta values for the countries studied:

Table 4: Significance and Impact of Each Cognitive Activation Practice for Enhancing Mathematical Reasoning on the Variance of Mathematical Literacy Among Students in Each of the Four Countries.

Cognitive Activation Practices	KSA		Singapore		UAE		USA	
	B (s.e)	*Beta*	*B (s.e)*	*Beta*	*B (s.e)*	*Beta*	*B (s.e)*	*Beta*
The teacher asked us to solve mathematical problems without performing calculations.	-1.07(0.99)	-0.02	-7.31(1.43)	-0.09	-2.48(0.78)	-0.03	-1.91(2.01)	-0.03
The teacher requested that we explain our problem-solving process for a mathematical question.	9.73(0.89)	0.21	3.03(1.50)	0.04	17.13(0.99)	0.22	6.20(1.76)	0.08
The teacher asked us to explain the assumptions we made when approaching a mathematical problem.	6.86(0.99)	0.14	-4.48(1.38)	-0.06	8.51(0.93)	0.11	-6.47(1.54)	-0.09
The teacher required us to articulate our reasoning when solving a mathematical problem.	7.29(0.92)	0.15	4.32(1.47)	0.05	18.86(1.00)	0.24	3.07(1.69)	0.04
The teacher asked us to defend our answers to mathematical questions.	3.09(0.93)	0.07	-6.73(1.33)	-0.09	7.84(0.85)	0.10	-2.11(1.74)	-0.03
The teacher encouraged us to consider the connections between new and previously learned mathematical concepts.	4.52(1.03)	0.09	0.57(1.50)	0.01	1.29(1.01)	0.02	-2.27(1.70)	-0.03
The teacher encouraged us to explore alternative problem-solving methods beyond those demonstrated in class.	6.26(0.87)	0.14	-0.32(1.46)	0.004	4.53(1.12)	0.06	-7.82(1.68)	-0.12
The teacher instructed us to persevere when encountering difficulties in mathematical tasks.	7.47(0.85)	0.16	7.53(1.56)	0.08	15.0)0.95)	0.19	6.27(1.89)	0.09
The teacher taught us to memorize rules and apply them in solving mathematical problems.	8.83(0.87)	0.19	6.07(1.92)	0.06	14.11(1.06)	0.17	10.94(1.93)	0.14

Table 4 reveals the significance and impact of each of the nine cognitive activation practices for enhancing mathematical reasoning on

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the variance in mathematical literacy across the four countries. The practice of "asking students to solve mathematics problems without calculation" showed statistically significant predictive effects in all four countries. Beta values were 0.02 for KSA, 0.09 for Singapore, and 0.03 for both the UAE and the USA. However, the B values indicated that this practice negatively contributes to mathematical literacy in all four countries, suggesting that increased use of this practice is associated with lower levels of mathematical literacy.

Additionally, the practice of "asking students to explain how they solved a mathematical problem" demonstrated statistically significant predictive effects across all countries. Beta values were 0.21 for KSA, 0.04 for Singapore, 0.22 for the UAE, and 0.08 for the USA. Moreover, the B values showed that this practice positively contributes to mathematical literacy across all four countries, indicating that increased use of this practice is associated with higher levels of mathematical literacy.

The practice of "asking students to explain the assumptions they made when solving a mathematical problem" showed statistically significant predictive effects across all countries. Beta values were 0.14 for KSA, -0.06 for Singapore, 0.11 for the UAE, and -0.09 for the USA. Notably, the B values indicated that this practice contributes positively to mathematical literacy in Saudi Arabia and the UAE, but negatively in Singapore and the USA.

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The practice of "asking students to explain their reasoning when solving a mathematical problem" demonstrated statistically significant predictive effects across all countries. Beta values were 0.15 for Saudi Arabia, 0.05 for Singapore, 0.24 for the UAE, and 0.04 for the USA. The B values showed that this practice positively contributes to mathematical literacy in all four countries.

The results of the multiple regression analysis revealed complex relationships between cognitive activation practices and mathematical literacy across the four countries studied. The practice of asking students to defend their mathematical answers demonstrated statistically significant predictive effects in all countries, with positive contributions to mathematical literacy in KSA ($\beta = 0.07$) and the UAE ($\beta = 0.10$), but negative effects in Singapore ($\beta = -0.09$) and the USA ($\beta = -0.03$).

Additionally, the practice of encouraging students to connect new and old mathematical concepts showed a positive effect on mathematical literacy in KSA ($\beta = 0.09$), Singapore ($\beta = 0.01$), and the UAE ($\beta = 0.02$), but a slight negative effect in the USA ($\beta = -0.03$).

Moreover, the practice of encouraging alternative problem-solving methods revealed mixed results. It had a positive effect in KSA ($\beta = 0.14$) and the UAE ($\beta = 0.06$), a minimal positive effect in Singapore ($\beta = 0.004$), but a negative effect in the USA ($\beta = -0.12$).

The practice of encouraging students to persevere through mathematical difficulties showed a statistically significant predictive effect in all four countries. Beta values were 0.16 for Saudi Arabia, 0.08

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for Singapore, 0.19 for the UAE, and 0.09 for the USA. The B values indicated that this practice contributes positively to mathematical literacy in all four countries.

Lastly, the practice of "teaching students to memorize rules and apply them in solving mathematical problems" demonstrated a statistically significant predictive effect across all countries. Beta values were 0.19 for Saudi Arabia, 0.06 for Singapore, 0.17 for the UAE, and 0.14 for the USA. The B values showed that this practice contributes positively to mathematical literacy in all four countries.

Discussion

The findings indicated that the level of cognitive activation practices is lowest in KSA compared to the other four countries in the study. This finding aligns with the results of Al-Sarhani (2024) and Al-Ghaith and Al-Ruwais (2022), who found low to moderate practices among mathematics teachers in the cities of Al-Jouf and Riyadh, particularly in practices related to helping students to reflect on and deepen their mathematical knowledge.

This explains the level of mathematical literacy among Saudi students compared to the other four countries. The average mathematical literacy score for Saudi students was 389, compared to 575 in Singapore, 431 in the UAE, and 465 in the USA (OECD, 2023).

However, the results from the UAE demonstrated some contradictions. While cognitive activation practices were highest in the UAE, the level of mathematical literacy among UAE students was lower

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than in the USA and Singapore. This can be attributed to the fact that mathematical literacy, like other subjects, is influenced by several factors (Al-Harbi, 2020). Therefore, caution must be carried out when interpreting the results causally.

The findings of this research align with the recommendations provided by the National Council of Teachers of Mathematics regarding effective teaching practices. Facilitating effective dialogue when students are asked to explain how they solved a problem and articulate their reasoning during problem-solving had a positive contribution to developing mathematical literacy across all four countries. Additionally, this study found that supporting productivity is an effective activation practice, as persevering through difficulties in mathematical tasks showed a positive contribution across all countries. The findings also highlighted the importance of procedural fluency, as memorizing and applying rules in solving mathematical problems contributed positively to developing mathematical literacy across all four countries.

Despite the statistically significant predictive effect of all items, some items contributed negatively to mathematical literacy in some countries and positively in others. This result is similar to the findings of Tourón et al. (2019), which revealed some contradictions requiring deeper investigation.

Overall, the results indicated that most cognitive activation practices contribute positively to mathematical literacy across the four countries. This aligns with the findings of other studies on cognitive

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activation practices, such as Yi & Lee (2017), Zhang et al. (2021), Li et al. (2021), Herreras (2017), Liu et al. (2022), Demir (2018), and Le Donné et al. (2016), which demonstrated that cognitive activation, as an important dimension of teaching quality, predicts student achievement and is positively associated with student performance in mathematics.

This result can be interpreted based on the study by Förtsch, Werner, Kotzebue, & Neuhaus (2016), which emphasized that teachers who adopt cognitive activation teaching provide students with opportunities to develop a deep understanding of content and actively engage in higher-order thinking through various learning activities, such as working on challenging tasks, connecting new concepts to prior knowledge, and stimulating scientific inquiry. Cantley et al. (2017) affirmed that collaborative teaching strategies for cognitive activation align with the fundamental principles of mathematics.

Conclusion and Recommendations

This study examined the relationship between cognitive activation practices and mathematical literacy across four countries. The findings suggested that while cognitive activation practices generally contribute positively to mathematical literacy, their effectiveness varies across different educational contexts. In this context, the study recommends the implementation of comprehensive training programs for mathematics teachers, focusing on cognitive activation practices and effective teaching strategies as outlined by NCTM.

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This study has provided a general description through responses and results from international assessments, which primarily serves as a descriptive foundation for future research. Consequently, this study recommends developing a deeper understanding of the phenomenon within specific contexts. For instance, future research could examine the national context in Saudi Arabia more specifically, attempting to reach individuals from the sample with varying results to derive logical explanations. These explanations can then serve as a foundation for informed decision-making.

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